Question

Energy Bands: Phosphorus is present in a germanium sample. Assume that one of its five valence electrons revolves in a Bohr orbit around each P^+ ion in the germanium lattice. (a) If the effective mass of the electron is 0.17 m_e and the dielectric constant of germanium is 16, find the radius of the first Bohr orbit of the electron. (b) The energy gap between the valence and conduction bands in germanium is 0.65 eV. How does the ionization energy of the above electron compare with this energy and with kT at room temperature?

Solution

(a) Adapting the formula for the radius of a Bohr orbit (eq 4.13 in the text)

$$r = \frac{h^2 \epsilon_r \epsilon_0}{\pi m^* e^2}$$

Where ϵ_r is the dielectric constant for Ge and m^* the effective mass for the 'outer' electron of P^+.

Hence

$$\begin{aligned} r &= \frac{(6.626 \times 10^{-34})^2 (16 \times 8.854 \times 10^{-12})}{\pi (0.17 \times 9.1095 \times 10^{-31})(1.602 \times 10^{-19})^2} \\ &= \frac{6.22 \times 10^{-77}}{1.25 \times 10^{-68}} \\ &= 4.98 \times 10^{-9}\, m \end{aligned}$$

The radius of the first Bohr orbit of the electron is 5.0 $\times 10^{-9}\, m$.

(b) The Bohr energy for this electron (eq 4.15 in the text)

$$\begin{aligned} E &= -\frac{e^2}{8\pi \epsilon_r \epsilon_0 r} \\ &= \frac{-(1.602 \times 10^{-19})^2}{8\pi (16 \times 8.854 \times 10^{-12})(4.98 \times 10^{-9})} \\ &= -\frac{2.57 \times 10^{-38}}{1.77 \times 10^{-17}} \\ &= -1.45 \times 10^{-21}\, J \quad \text{or} \quad -9.0 \times 10^{-3}\, eV \end{aligned}$$

This ionization energy of 0.009 eV is much smaller than the energy gap of 0.65 eV between the valence and conduction bands in Ge.

kT at 20°C is 0.025 eV ($8.617 \times 10^{-5} \times 293$) and is of the same order of magnitude as the ionization energy.
