Question

Shell Model: A simplified model of the deuteron consists of a neutron and a proton in a square potential well 2 fm in radius and 35 MeV deep. Is this model consistent with the uncertainty principle?

Solution

Uncertainty principle

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Since $\Delta x = 2\, fm$

$$\begin{aligned}
\Delta p &\geq \frac{\hbar}{2(2 \times 10^{-15})} \\
&\geq \frac{6.582 \times 10^{-16}}{2(2 \times 10^{-15})} \\
&\geq 0.1646\ eVs/m
\end{aligned}$$

$$\begin{aligned}
\Delta E &\geq \frac{p^2}{2m} \\
&\geq \frac{(0.1646)^2}{2(1.008\, u)} \\
&\geq \frac{0.0271}{2(1.008)} \frac{(3 \times 10^8)^2}{(931.49 \times 10^3)} \\
&\geq \frac{2.440 \times 10^{15}}{1.878 \times 10^9} \\
&\geq 1.30 \times 10^6\, eV \quad \text{or} \quad 1.30\, MeV
\end{aligned}$$

The uncertainty principle implies a kinetic energy for the neutron/proton of the order $1.30\, MeV$ which is consistent with a $35\, MeV$ potential well model.
