Question

Some Nuclear Properties: A system of a million distinguishable protons is in thermal equilibrium at 20°C in a 1.00 T magnetic field. More of the protons are in the lower energy spin-up state than in the higher energy spin-down state. (a) On the average, how many more? (b) Repeat the calculation for a temperature of $20K$. (c) What do these results suggest about how strongly such a system will absorb *em* radiation at the Lamour frequency? (d) Could such a system in principle be used as the basis of a laser? If not, why not?

Solution

(a) For distinguishable particles energy ϵ at temperature T (eq 9.3 in the text)

$$n(\epsilon) = Ag(\epsilon)e^{-\epsilon/kT} \qquad \text{where} \qquad g(\epsilon) = \text{no. of states energy}\,\epsilon \quad (1 \text{ for each state in this case})$$

Hence

$$\frac{n(\epsilon_2)}{n(\epsilon_1)} = \frac{g(\epsilon_2)}{g(\epsilon_1)}\, e^{-(\epsilon_2-\epsilon_1)/kT}$$
$$\frac{n(\downarrow)}{n(\uparrow)} = e^{-\Delta E/kT}$$

Where ΔE, energy difference between the two states is given by

$$\Delta E = 2\mu_{pz}B = (2)(2.793)(3.152\times10^{-8})(1.00) = 1.761\times10^{-7}\,eV$$

$$\frac{n(\downarrow)}{n(\uparrow)} = e^{-(1.761\times10^{-7})/(8.617\times10^{-5})(293)}$$
$$n(\downarrow) = 0.999993\, n(\uparrow) \qquad (1)$$

$$n(\uparrow) + n(\downarrow) = 10^6 \qquad (2)$$
$$1.999993\, n(\uparrow) = 10^6$$
$$n(\uparrow) = \frac{10^6}{2(1-0.0000035)}$$
$$= 10^6(1.0000035)/2 = 500001.75$$

Hence, on average, there are 3.5 (2×1.75) more protons in the spin-up state than the spin-down state.

(b) At 20 K

$$\frac{n(\downarrow)}{n(\uparrow)} = e^{-(1.761\times10^{-7})/(8.617\times10^{-5})(20)}$$
$$n(\downarrow) = 0.999898\, n(\uparrow) \qquad (1b)$$
$$n(\uparrow) = \frac{10^6}{2(1-0.0000051)}$$
$$= 10^6(1.0000051)/2 = 500025.5$$

At this temperature there are 51 protons more in the spin-up state.

(c) Since the populations are so close the rates of stimulated emission and of absorption will be nearly equal i.e. little net absorption. The populations are even closer at higher temperatures so even less net absorption.

(d) A minimum of three energy levels is necessary for lasers, this two-level system cannot be used.
