Question

Half-Life: The heaviest nuclei are probably created in supernova explosions and become distributed in galactic matter from which later stars (and their planets) form. Under the assumption that equal amounts of the ^{235}U and ^{238}U now in the earth were created in this way in the same supernova, calculate how long ago this occurred from their respective observed abundances of 0.7 and 99.3 percent and respective half-lives of 7.0 $\times 10^8\, y$ and 4.5 $\times 10^9\, y$.

Solution

The number N of undecayed nuclei at time t is $N = N_0 e^{-\lambda t}$ where λ is the decay constant (related the the half-life, $T_{1/2} = ln\, 2/\lambda = 0.693/\lambda$). Hence

$\lambda_{235} = 0.693/(7.0 \times 10^8\, y) = 9.90 \times 10^{-10}\, /y$
$\lambda_{238} = 0.693/(4.5 \times 10^9\, y) = 1.54 \times 10^{-10}\, /y$

Assuming N_0 is equal for both isotopes and taking t as the time since the supernova

$$\begin{aligned} \frac{N_{235}}{N_{238}} &= \frac{e^{-\lambda_{235} t}}{e^{-\lambda_{238} t}} \\ \frac{0.7}{99.3} &= e^{-(\lambda_{235} - \lambda_{238})t} \\ 7.049 \times 10^{-3} &= e^{-(8.36 \times 10^{-10})t} \\ (8.36 \times 10^{-10})t &= 4.955 \\ t &= 5.927 \times 10^9\, y \end{aligned}$$

Under this assumption the supernova explosion must have occurred 5.9×10^9 years ago.