Question

Nuclear Reactions: A particle of mass m_1 and kinetic energy KE_1 collides head on with a stationery particle of mass m_2. The two particles then move apart with the target particle having the kinetic energy KE_2'. (a) Use conservation of momentum and conservation of kinetic energy in a non-relativistic calculation to show that $KE_2'/KE_1 = 4(m_2/m_1)/(1+m_2/m_1)^2$. (b) What percentage of its initial KE does a neutron lose when it collides head on with a proton? With a deuteron? With a ^{12}C nucleus? With a ^{235}U nucleus?

Solution

a) Conservation of momentum

$$m_1v_1 = m_1v_1' + m_2v_2' \quad \rightarrow \quad v_1' = \frac{m_1v_1 - m_2v_2'}{m_1}$$

Conservation of energy

$$\begin{aligned} KE_1 &= \tfrac{1}{2}m_1{v_1'}^2 + KE_2' \\ &= \tfrac{1}{2}m_1\left(\frac{m_1v_1 - m_2v_2'}{m_1}\right)^2 + KE_2' \\ &= \tfrac{1}{2}\frac{m_1^2v_1^2}{m_1} - \frac{m_1m_2v_1v_2'}{m_1} + \tfrac{1}{2}\frac{m_2^2{v_2'}^2}{m_1} + KE_2' \\ &= KE_1 - m_2(v_1v_2') + \frac{m_2}{m_1}KE_2' + KE_2' \\ KE_1 &= KE_1 - 2m_2\sqrt{\frac{KE_1KE_2'}{m_1m_2}} + \frac{m_2}{m_1}KE_2' + KE_2' \\ 2m_2\sqrt{\frac{KE_1KE_2'}{m_1m_2}} &= KE_2'\left(1 + \frac{m_2}{m_1}\right) \\ 4\,\frac{m_2}{m_1}\,KE_1 &= KE_2'\left(1 + \frac{m_2}{m_1}\right)^2 \\ KE_2'/KE_1 &= 4(m_2/m_1)/(1 + m_2/m_1)^2 \end{aligned}$$

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b) Neutron collides with proton $\quad m_2/m_1 = 1.007/1.009 = 0.9980$

$$KE_2'/KE_1 = 4(.9980)/(1.998)^2 = 1.000 \quad \text{i.e. } 100\% \text{ lost.}$$

Neutron collides with deuteron $\quad m_2/m_1 = 2.014/1.009 = 1.996$

$$KE_2'/KE_1 = 4(1.996)/(2.996)^2 = 0.8895 \quad \text{i.e. } 89\% \text{ lost.}$$

Neutron collides with ^{12}C nucleus $\quad m_2/m_1 = 12/1.009 = 11.89$

$$KE_2'/KE_1 = 4(11.89)/(12.89)^2 = 0.2863 \quad \text{i.e. } 29\% \text{ lost.}$$

Neutron collides with ^{235}U nucleus $\quad m_2/m_1 = 235.0/1.009 = 232.9$

$$KE_2'/KE_1 = 4(232.9)/(233.9)^2 = 0.0170 \quad \text{i.e. } 1.7\% \text{ lost.}$$