Question

Hadrons: Find the energy of the photon emitted in the decay $\Sigma^0 \to \Lambda^0 + \gamma$.

Solution

Taking c=1 and given $m_{\Sigma^0} = 1193\,MeV/c^2 \quad m_{\Lambda^0} = 1116\,MeV/c^2$ (Table 13.3)

Conservation of momentum

$$m_{\Lambda^0} v_{\Lambda^0} = p_\gamma = E_\gamma \quad \to \quad v_{\Lambda^0} = E_\gamma / m_{\Lambda^0}$$

Conservation of energy

$$\begin{aligned}
m_{\Sigma^0} &= m_{\Lambda^0} + \tfrac{1}{2} m_{\Lambda^0} v_{\Lambda^0}^2 + E_\gamma \\
&= m_{\Lambda^0} + \tfrac{1}{2} m_{\Lambda^0} \left(\frac{E_\gamma}{m_{\Lambda^0}} \right)^2 + E_\gamma \\
1193 &= 1116 + \tfrac{1}{2}(1116) \left(\frac{E_\gamma}{1116} \right)^2 + E_\gamma \\
77 &= E_\gamma^2 / 2232 + E_\gamma \\
E_\gamma^2 + 2232 E_\gamma - 171864 &= 0 \\
E_\gamma &= \frac{-2232 \pm \sqrt{2232^2 + 4(171864)}}{2} \\
&= \frac{-2232 \pm 2381}{2} \\
&= 74.5\,MeV \qquad \text{(ignoring -ve root)}
\end{aligned}$$

The energy of the photon emitted in the decay is $74.5\,MeV$.