Question

Mass and Momentum: In its own frame of reference, a proton takes 5 *min* to cross the Milky Way galaxy, which is about 10^5 *light-years* in diameter. (a) What is the approximate energy of the proton in electron volts? (b) About how long would a proton take to cross the galaxy as measured by an observer in the galaxy's reference frame?

Solution

a) In its own frame, the distance travelled by the proton is $L = L_0\sqrt{1 - v^2/c^2}$ where L_0 is the diameter of the Milky Way and $v = L/t,\ t$ being the 5 *min* taken to cross it.

$$\begin{aligned}
L &= L_0\sqrt{1 - v^2/c^2} \\
&= L_0\sqrt{1 - L^2/c^2t^2} \\
L^2 &= L_0^2(1 - L^2/c^2t^2) \\
L^2 &= \frac{L_0^2}{1 + L_0^2/c^2t^2} \\
&= \frac{c^2t^2}{1 + c^2t^2/L_0^2} \\
L &= ct\left(1 + \frac{c^2t^2}{L_0^2}\right)^{-1/2} \\
&\approx ct\left(1 - \frac{c^2t^2}{2L_0^2} + ...\right) \\
&= ct\left(1 - \frac{300^2}{2(60 \times \times 60 \times 24 \times 365 \times 10^5)^2}\right) \\
&= ct(1 - 4.525 \times 10^{-21})
\end{aligned}$$

Hence the velocity of the proton is $c(1 - 4.525 \times 10^{-21})$ i.e. near light speed.

Energy of proton

$$\begin{aligned}
\gamma &= \frac{1}{\sqrt{1 - v^2/c^2}} \\
&= \frac{1}{\sqrt{1 - (1 - 4.525 \times 10^{-21})^2}} \\
&\approx 1/\sqrt{9.05 \times 10^{-21}} \\
&= 1.051 \times 10^{10} \\
E &= \gamma mc^2 \\
&= 1.051 \times 10^{10} \times 938.28\, MeV \\
&= 9.863 \times 10^{18}\, eV
\end{aligned}$$

The total energy of the proton is approximately $10^{19}\, eV$

b) Time taken for proton to cross in the galaxy's frame is $(1 - 4.525 \times 10^{-21})10^5$ *light-years* i.e. about the same as a photon or 10^5 *light-years*.