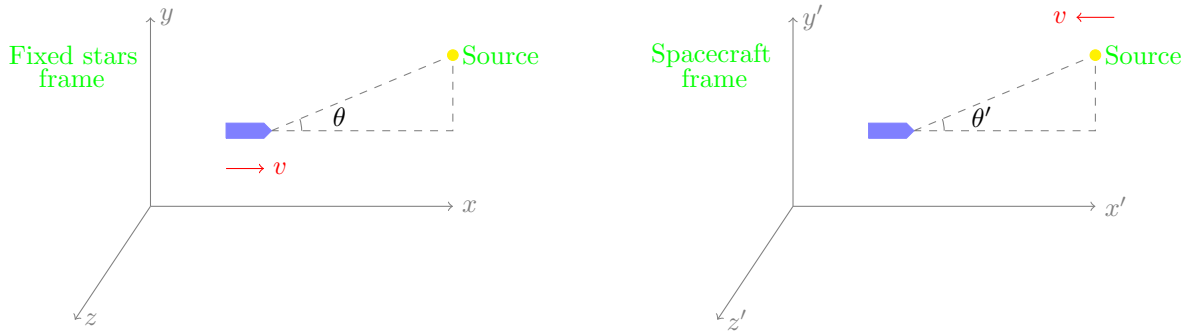
Question

Lorentz Transformation: A spacecraft moving in the $+x$ direction receives a light signal from a source in the xy plane. In the reference frame of the fixed stars, the speed of the spacecraft is v and the signal arrives at an angle θ to the axis of the spacecraft. (a) With the help of the Lorentz transformation find the angle θ' at which the signal arrives in the reference frame of the spacecraft. (b) What would you conclude from this result about the view of the stars from a porthole on the side of the spacecraft?

Solution



a) In the fixed frame the initial distance from spacecraft to source is $x/cos\theta$ hence in that frame $t = x/c\,cos\theta$. Thus using the Lorentz transformation:

$$\begin{aligned} x' &= \frac{x-(-v)t}{\sqrt{1-(-v)^2/c^2}} \\ &= \frac{x+v(x/c\,cos\theta)}{\sqrt{1-v^2/c^2}} \\ y' &= y \end{aligned}$$

$$\begin{aligned} tan\,\theta' &= \frac{y'}{x'} \\ &= \frac{y\sqrt{1-v^2/c^2}}{x+v(x/c\,cos\theta)} \\ &= \frac{tan\theta\sqrt{1-v^2/c^2}}{1+v/c\,cos\theta} \\ &= \frac{sin\theta\sqrt{1-v^2/c^2}}{cos\theta+v/c} \\ \theta' &= tan^{-1}\left(\frac{sin\theta\sqrt{1-v^2/c^2}}{cos\theta+v/c}\right) \end{aligned}$$

b) As $v \to c \quad \theta' \to 0$. i.e. The light source (and hence stars in front of spacecraft) appear(s) further forward in the field of view than if v were zero.