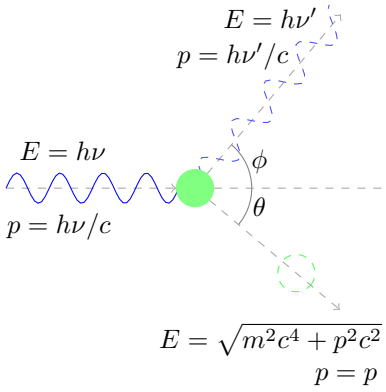
Question

Compton Effect: A photon whose energy equals the rest energy of the electron undergoes a Compton collision with an electron. If the electron moves off at an angle of 40° with the original photon direction, what is the energy of the scattered photon?

Solution

Conservation of horizontal and vertical momenta give

$$\begin{aligned} h\nu' cos\phi &= h\nu - pc\,cos\theta \\ h\nu' sin\phi &= pc\,sin\theta \\ \rightarrow \quad (h\nu')^2 &= (h\nu - pc\,cos\theta)^2 + (pc\,sin\theta)^2 \\ (h\nu')^2 &= (h\nu)^2 - 2h\nu pc\,cos\theta + p^2c^2 \\ 2h\nu pc\,cos\theta &= (h\nu)^2 - (h\nu')^2 + p^2c^2 \qquad (1) \end{aligned}$$



The KE gained by the electron is $h\nu - h\nu'$ and the initial energy of the photon is $h\nu = mc^2$. Thus equating the expressions for the total energy of the electron and substituting

$$\begin{aligned} KE + mc^2 &= \sqrt{m^2c^4 + p^2c^2} \\ (KE + mc^2)^2 &= m^2c^4 + p^2c^2 \\ p^2c^2 &= KE^2 + 2mc^2\,KE \\ p^2c^2 &= (h\nu - h\nu')^2 + 2h\nu(h\nu - h\nu') \qquad (2) \end{aligned}$$

Subs for p^2c^2 in (1) and simplifying

$$\begin{aligned} 2h\nu pc\,cos\theta &= (h\nu)^2 - (h\nu')^2 + (h\nu - h\nu')^2 + 2h\nu(h\nu - h\nu') \\ 2h\nu pc\,cos\theta &= 4(h\nu)^2 - 4(h\nu)(h\nu') \\ pc\,cos\theta &= 2(h\nu - h\nu') \\ p^2c^2\,cos^2\theta &= 4(h\nu - h\nu')^2 \end{aligned}$$

Subs for p^2c^2 once more and simplifying gives equation (3) for $h\nu'$ (energy of scattered photon)

$$\big((h\nu - h\nu')^2 + 2h\nu(h\nu - h\nu')\big)cos^2\theta = 4(h\nu - h\nu')^2$$

$$(cos^2\theta - 4)(h\nu')^2 + 4h\nu(2 - cos^2\theta)(h\nu') + (h\nu)^2(3cos^2\theta - 4) = 0 \qquad (3)$$

$$\begin{aligned} h\nu' &= \frac{-4h\nu(2 - cos^2\theta) \pm \sqrt{16(h\nu)^2(2 - cos^2\theta)^2 - 4(cos^2\theta - 4)(h\nu)^2(3cos^2\theta - 4)}}{2(cos^2\theta - 4)} \\ &= h\nu\left(\frac{-2(2 - cos^2\theta) \pm \sqrt{4(2 - cos^2\theta)^2 - (cos^2\theta - 4)(3cos^2\theta - 4)}}{cos^2\theta - 4}\right) \end{aligned}$$

Subs $\theta = 40°$ and $h\nu = 511\,keV$ (initial energy of photon)

$$h\nu' = 511\left(\frac{-2.826 \pm \sqrt{0.3463}}{-3.413}\right) = 511 \times 0.656 = 335 \qquad \textit{(taking sensible root)}$$

The energy of the scattered photon is $335\,keV$