Question

De Broglie Waves: By what percentage will a non-relativistic calculation of the de Broglie wavelength of a 100 keV electron be in error?

Solution

Classical case:

$$\begin{aligned} KE &= 10^5\, eV = 10^5/(6.242 \times 10^{18})\, J \\ &= 1.602 \times 10^{-14}\, J \end{aligned}$$

$$\begin{aligned} mv &= \sqrt{2m\, KE} \\ &= \sqrt{2(9.1095 \times 10^{-31})(1.602 \times 10^{-14})} \\ &= 1.7084 \times 10^{-22}\, kg\, m/s \end{aligned}$$

$$\begin{aligned} \lambda &= \frac{h}{mv} \\ &= \frac{6.626 \times 10^{-34}}{1.7084 \times 10^{-22}}\, m \\ &= 3.878 \times 10^{-12}\, m \end{aligned}$$

Relativistic case:

$$\begin{aligned} E = KE + mc^2 &= \sqrt{p^2c^2 + m^2c^4} \\ 100 &= \sqrt{p^2c^2 + 511^2} - 511 \\ p^2c^2 &= (611^2 - 511^2) \\ pc &= 335.0\ keV/c \end{aligned}$$

$$\begin{aligned} \lambda &= \frac{h}{p} \\ &= \left(\frac{4.136 \times 10^{-15}}{3.35 \times 10^5}\right)\, (3 \times 10^8)\, m \\ &= 3.702 \times 10^{-12}\, m \end{aligned}$$

Hence the percentage error in λ

$$\begin{aligned} &= \frac{3.878 - 3.702}{3.702} \times 100 \\ &= 4.754\,\% \end{aligned}$$

The non-relativistic error in the calculation of the de Broglie wavelength would be about 4.8%.