Question

Nuclear motion: (a) Derive a formula for the energy levels of a **hydrogenic atom**, which is an ion such as He^+ or Li^{2+} whose nuclear charge is $+Ze$ and which contains a single electron. (b) Sketch the energy levels of the He^+ ion and compare them with the energy levels of the H atom. (c)An electron joins a bare helium nucleus to form a He^+ ion. Find the wavelength of the photon emitted in this process if the electron is assumed to have had no kinetic energy when it combined with the nucleus.

Solution

a) Energy levels for a H atom $(KE + PE)$

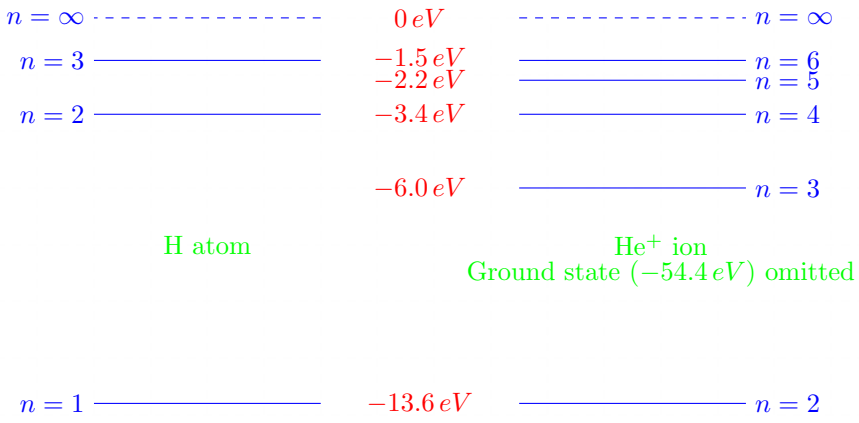
$$E_n = -\frac{e^2}{8\pi\epsilon_0 r_n} \qquad \text{where} \qquad r_n = \frac{n^2h^2\epsilon_0}{\pi m e^2}$$

Hence for a hydrogenic atom (nuclear charge $+Ze$)

$$E_n = -\frac{Ze^2}{8\pi\epsilon_0 r_n} \qquad \text{where} \qquad r_n = \frac{n^2h^2\epsilon_0}{\pi m Ze^2}$$

$$E_n = -\frac{mZ^2e^4}{8\epsilon_0^2h^2}\left(\frac{1}{n^2}\right)$$

b) Energy levels in H and He^+



c) Energy level for ground state of He^+

$$\begin{aligned} E_1 &= -\frac{m2^2e^4}{8\epsilon_0^2h^2}\left(\frac{1}{1^2}\right) \\ &= -54.4\,eV \end{aligned}$$

$$\begin{aligned} hc/\lambda &= E_1 \\ \lambda &= \frac{(4.136\times10^{-15})(3\times10^8)}{54.4} \\ &= 2.28\times10^{-8} \end{aligned}$$

An electron $(0\ KE)$ joining a bare He nucleus will emit a photon of wavelength $2.28\times10^{-8}\,m$.