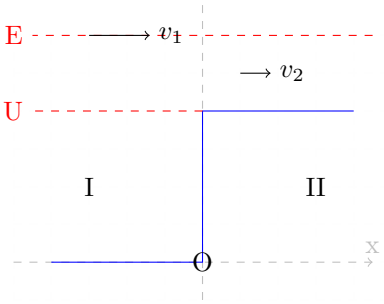
Question

The Tunnel Effect: Consider a beam of particles of kinetic energy E incident on a potential step at $x = 0$, that is U high, where $E > U$. Schrödinger's equation gives the solutions:

$$\psi_I = Ae^{ik_1 x} + Be^{-ik_1 x} \qquad \psi_{II} = Ce^{ik_2 x} + De^{-ik_2 x}$$



(a) Explain why $D = 0$. (b) Show that the transmission probability here is $T = CC^* v_2 / AA^* v_1 = 4k_1^2/(k_1 + k_2)^2$. (c) A $1.00\,mA$ beam of electrons moving at $2.00 \times 10^6\, m/s$ enters a region with a sharply defined boundary in which the electron speeds are reduced to $1.00 \times 10^6\, m/s$ by a difference in potential. Find the transmitted and reflected currents.

Solution

a) $De^{-ik_2 x}$ has no physical meaning since this represents a wave travelling in the $-ve\, x$ direction in region II. i.e. $D = 0$

b) At the boundary

$$\psi_I = \psi_{II} \qquad \rightarrow \qquad A + B = C$$

$$\frac{d\psi_I}{dx} = \frac{d\psi_{II}}{dx} \qquad \rightarrow \qquad Aik_1 - Bik_1 = Cik_2$$

Hence

$$\begin{aligned} Aik_1 - (C - A)ik_1 &= Cik_2 \\ 2Ak_1 - Ck_1 &= Ck_2 \\ \frac{A}{C} &= \frac{k_1 + k_2}{2k_1} \end{aligned}$$

The transmission probability is

$$\begin{aligned} T &= \frac{CC^* v_2}{AA^* v_1} \\ &= \frac{4k_1^2}{(k_1 + k_2)^2} \frac{v_2}{v_1} \qquad * \end{aligned}$$

c) Taking wave number $k = 2\pi(mv)/h \quad \rightarrow \quad k_1/k_2 = v_1/v_2 = 2$ hence

$$\begin{aligned} T &= \frac{4 \times 2^2}{(2+1)^2} \left(\frac{1}{2}\right) \\ &= \frac{8}{9} \end{aligned}$$

Current transmitted is $\frac{8}{9} \times 10^{-3} = .889\, Ma$

Current reflected is $\frac{1}{9} \times 10^{-3} = .111\, Ma$

** There is either a misprint in the question or I've done this wrong!*