Question

Spin-Orbit Coupling: The spin-orbit effect splits the $3P \rightarrow 3S$ transition in sodium into two lines, 589.0 nm corresponding to $3P_{3/2} \rightarrow 3S_{1/2}$ and 589.6 nm corresponding to $3P_{1/2} \rightarrow 3S_{1/2}$. Use these wavelengths to calculate the effective magnetic field experienced by the outer electron in the sodium atom as a result of its orbital motion.

Solution

Spin-orbit coupling splits the $3p$ state of the sodium atom into two substates ΔE apart; depending on the orientation of its spin vector $\mathbf{S}$ the energy of the atomic electron in this state will be higher or lower by $\mu_B B$ where the Bohr magneton $\mu_B = e\hbar/2m$ and B is the effective magnetic field due to the atomic nucleus, hence

$$\begin{aligned}\Delta E &= 2\,\frac{e\hbar}{2m}\,B \\ &= \frac{eh}{2\pi m}\,B\end{aligned}$$

$$B = \frac{2\pi m \Delta E}{eh}$$

The difference in energy levels corresponds to the difference in wavelengths for the two transitions

$$\begin{aligned}\nu &= \frac{c}{\lambda} \\ \Delta\nu &= \frac{c}{\lambda^2}\,\Delta\lambda \quad \text{(ignoring -ve sign)}\end{aligned}$$

$$\begin{aligned}\Delta E &= h\Delta\nu \\ &= \frac{hc\Delta\lambda}{\lambda^2}\end{aligned}$$

$$\begin{aligned}B &= \frac{2\pi m}{eh}\,\frac{hc\Delta\lambda}{\lambda^2} \\ &= \frac{2\pi mc\,\Delta\lambda}{e\lambda^2} \\ &= \frac{2\pi(9.1095\times10^{-31})(3\times10^{8})(0.6\times10^{-9})}{(1.602\times10^{-19})(5.89\times^{-7})^2} \\ &= 18.61\end{aligned}$$

The effective magnetic field due to the atomic nucleus of sodium is $18.6\,T$