Question

Vibrational Energy Levels: The force constant of the $^1H^{19}F$ molecule is approximately $966\,N/m$. (a) Find the frequency of vibration of the molecule. (b) The bond length is approximately $0.92\,nm$. Plot the potential energy of this molecule versus internuclear distance in the vicinity of $0.92\,nm$ and show the vibrational energy levels.

Solution

a) Reduced mass of $^1H^{19}F$

$$m' = \frac{m_1 m_2}{m_1 + m_2} = \frac{19}{20}(1.66 \times 10^{-27}) = 1.58 \times 10^{-27}\,kg$$

Frequency of vibration

$$\begin{aligned} \nu_0 &= \frac{1}{2\pi}\sqrt{\frac{k}{m'}} \\ &= \frac{1}{2\pi}\sqrt{\frac{966}{1.58 \times 10^{-27}}} \\ &= 1.24 \times 10^{14} \end{aligned}$$

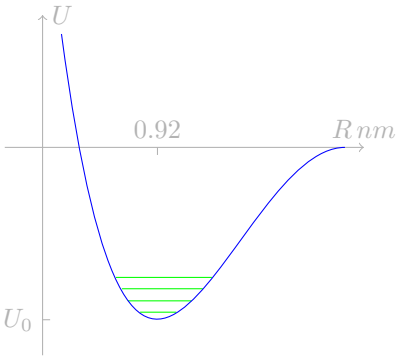
The frequency of vibration of $^1H^{19}F$ is $1.24 \times 10^{14}\,Hz$.

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b) Vibrational energy levels (above U_0)

$$E_v = (v + \tfrac{1}{2})\nu_0 \qquad v = 0,\, 1,\, 3\,...$$

$$\begin{aligned} E_0 &= \frac{h}{2}\nu_0 = 0.257\,eV \\ E_1 &= \frac{3h}{2}\nu_0 = 0.770\,eV \\ E_2 &= \frac{5h}{2}\nu_0 = 1.28\,eV \\ E_3 &= \frac{7h}{2}\nu_0 = 1.80\,eV \end{aligned}$$



This suggests a spacing between energy levels of approximately $0.513\,eV$. However for larger values of vibrational quantum number v, the parabolic approximation for the potential energy curve becomes less valid and the spacing between levels progressively smaller.
