Question ?

Planck Radiation Law: Considering the sun as a blackbody at 6000 K, estimate the proportion of its total radiation that consists of yellow light between 570 nm and 590 nm.

Solution

The total energy per unit volume of a photon gas using Planck's radiation formula

$$\begin{aligned}\epsilon_0 &= \int_0^\infty u(\nu)d\nu \\ &= \int_0^\infty \frac{8\pi h}{c^3}\frac{\nu^3 d\nu}{e^{h\nu/kT}-1}\end{aligned}$$

Using the substitution $x = h\nu/kT$ or $\nu = kTx/h$ $d\nu = kTdx/h$

$$\begin{aligned}\epsilon_0 &= \frac{8\pi k^4T^4}{h^3c^3}\int_0^\infty \frac{x^3dx}{e^x-1} \\ &= \frac{8\pi k^4T^4}{h^3c^3}(6.494) \quad *\end{aligned}$$

$\lambda = 570\,nm \quad \rightarrow \quad x = \dfrac{h\nu}{kT} = \dfrac{hc}{\lambda kT} = 4.209$

$\lambda = 590\,nm \quad \rightarrow \quad x = 4.066$

Hence the energy of 'yellow' photons per unit volume

$$\begin{aligned}\epsilon_1 &= \frac{8\pi k^4T^4}{h^3c^3}\int_{4.066}^{4.209} \frac{x^3dx}{e^x-1} \\ &= \frac{8\pi k^4T^4}{h^3c^3}(0.1643) \quad *\end{aligned}$$

Ratio of photon energies $\epsilon_1/\epsilon_0 = 0.1643/6.494 = 0.0253$.

Hence the proportion of the sun's total radiation that consists of yellow light is about 2.5 %.

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* These are standard integrals which can be looked up or calculated using a graphing utilty.