Question

Dying Stars: Consider a white dwarf star whose mass is half that of the sun and whose radius is 0.01 that of the sun. Assume it consists of completely ionized carbon atoms (mass 12 u), so that there are six electrons per nucleus, and its interior temperature is $10^7\,K$. (a) Find the Fermi energies of the carbon nucleus gas and of the electron gas. (b) Compare these energies with kT to see whether each gas is degenerate or nondegenerate.

Solution

The sun has a radius of $7.0 \times 10^8\,m$ and a mass of $2.0 \times 10^{30}\,kg$ (Ex 52 in the text), hence the radius and mass of the white dwarf are $7.0 \times 10^6\,m$ and $1.0 \times 10^{30}\,kg$ respectively.

The Fermi energy of each gas (eq 9.56 in the text)

$$\epsilon_F = \frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{2/3}$$

Where N/V is the density of the individual particles.

a) For the carbon nucleus gas

$$\begin{aligned}\frac{N}{V} &= \frac{(1.0 \times 1.0^{30})/\left(12(1.66 \times 10^{-27})\right)}{\frac{4}{3}\pi(7.0 \times 10^6)^3} \\ &= \frac{5.02 \times 10^{55}}{1.437 \times 10^{21}} \\ &= 3.494 \times 10^{34}\end{aligned}$$

$$\begin{aligned}\epsilon_{F_C} &= \frac{(6.626 \times 10^{-34})^2}{2 \times 12(1.66 \times 10^{-27})}\left(\frac{3}{8\pi}(3.494 \times 10^{34})\right)^{2/3} \\ &= (1.102 \times 10^{-41})(2.591 \times 10^{22}) \\ \epsilon_{F_C} &= 2.855 \times 10^{-19}\,J \quad \text{or} \quad 1.782\,eV\end{aligned}$$

For the electron gas $N/V = 6 \times 3.494 \times 10^{34} = 2.096 \times 10^{35}$

$$\begin{aligned}\epsilon_{F_e} &= \frac{(6.626 \times 10^{-34})^2}{2 \times (9.1095 \times 10^{-31})}\left(\frac{3}{8\pi}(2.096 \times 10^{35})\right)^{2/3} \\ &= (2.410 \times 10^{-37})(8.554 \times 10^{22}) \\ \epsilon_{F_e} &= 2.061 \times 10^{-14}\,J \quad \text{or} \quad 1.287 \times 10^5\,eV\end{aligned}$$

b) Comparing these Fermi energies with $kT = 816.7\,eV \quad (8.617 \times 10^{-5} \times 10^7)$

For the carbon nuclei $\epsilon_{F_C} << kT$ i.e. nondegenerate

For the electrons $\epsilon_{F_e} >> kT$ i.e. degenerate

In a white dwarf it is the pressure of the electron gas which balances the gravitational pull and prevents it from collapsing.