Problem

The first estimate of Newton's constant was made by the astronomer Nevil Maskelyne in 1774 by measuring the angle between the directions of the apparent plumb-line to the vertical on opposite sides of the Scottish mountain Schiehallion (height 1081m, chosen for its regular conical shape). Find a rough estimate of the angle through which a plumb-line is deviated by the gravitational attraction of the mountain, by modelling the mountain as a sphere of radius 500m and density 2.7×10^3 kg m^{-3}, and assuming that its gravitational effect is the same as though the total mass were concentrated at its centre.

Solution

The tangent of the required angle to the vertical is taken as the ratio of forces on the plumb bob from mountain and Earth respectively. Hence

$$\begin{aligned} tan\,\theta &= F_M/F_E \\ &= \frac{GM_M}{500^2} \div \frac{GM_E}{R_E^2} \\ &= \frac{\frac{4}{3}\pi\,500^3(2.7\times10^3)}{500^2} \times \frac{(6.371\times10^6)^2}{5.974\times10^{24}} \\ &= 5.655\times10^6 \times 6.794\times10^{-12} \\ &= 3.842\times10^{-5} \\ \theta &= (2.20\times10^{-3})^\circ \\ &= 7.92'' \end{aligned}$$

A rough estimate of the angle of deviation from the vertical is 7.9 *seconds*.
