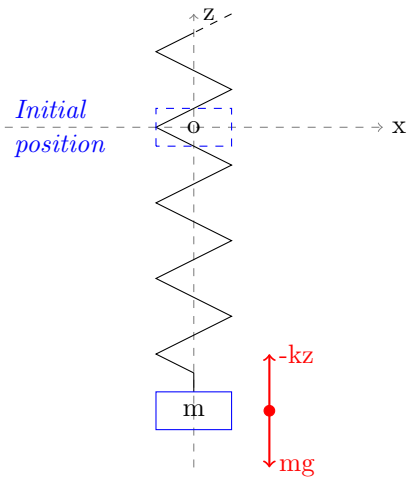
Problem

A weight of mass m is hung from the end of a spring which provides a restoring force equal to k times its extension. The weight is released from rest with the spring unextended. Find its position as a function of time, assuming negligible damping.



Solution

Taking the initial rest position at zero *i.e.* $z=0$ the **net** restoring force on the mass in the +ve z direction is $-kz-mg$ since z is always -ve after release. Thus the equation of motion is

$$\begin{aligned} -kz - mg &= m\ddot{z} \\ m\ddot{z} + k\left(z + \frac{mg}{k}\right) &= 0 \\ \ddot{z} + \omega^2\left(z + \frac{mg}{k}\right) &= 0 \end{aligned}$$

Where $\omega = \sqrt{\frac{k}{m}}$; and the solution is:

$$\begin{aligned} z + \frac{mg}{k} &= a cos(\omega t - \theta) \\ z &= a cos(\omega t - \theta) - \frac{mg}{k} \\ \dot{z} &= -aw sin(\omega t - \theta) \end{aligned}$$

Initial conditions t=0, $\dot{z} = 0 \Rightarrow \theta = 0$; t=0, z=0 $\Rightarrow a = \frac{mg}{k}$; so the final solution is:

$$z = -\frac{mg}{k}(1 - cos\omega t)$$

This is simple harmonic motion with angular frequency ω about the equilibrium position $z = -\frac{mg}{k}$

