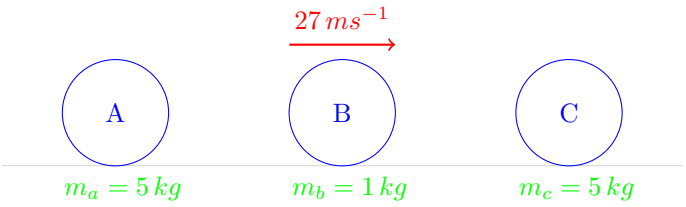
Problem

Three perfectly elastic bodies of masses 5 kg, 1 kg, 5 kg are arranged in order on a straight line, and are free to move along it. Initially, the middle one is moving with velocity 27 ms^{-1}, and the others are at rest. Find how many collisions take place in the subsequent motion, and verify that the final value of the kinetic energy is equal to the initial value.

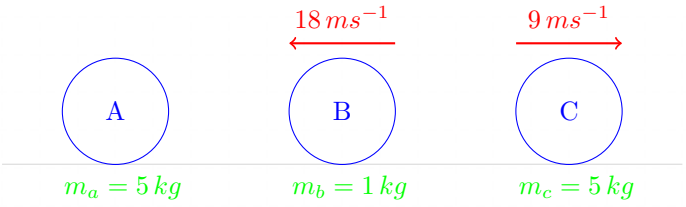
Solution

Collision 1 (B $\rightarrow$ C, where C is at rest) [Diagrams show conditions before collision]



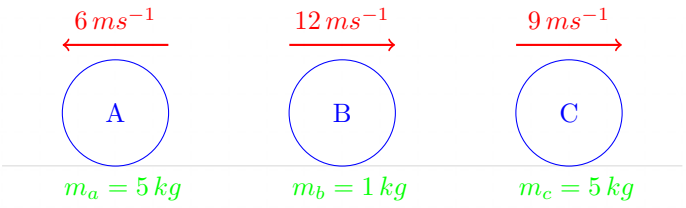
$$v_b = \frac{m_b - m_c}{m_b + m_c}\,27 = -\frac{4}{6}\,27 = -18\,ms^{-1}$$
$$v_c = \frac{2m_b}{m_b + m_c}\,27 = \frac{2}{6}\,27 = 9\,ms^{-1}$$

Collision 2 (B $\rightarrow$ A, where A is at rest)



$$v_b = \frac{m_b - m_a}{m_b + m_a}\,(-18) = -\frac{4}{6}\,(-18) = 12\,ms^{-1}$$
$$v_a = \frac{2m_b}{m_b + m_a}\,(-18) = \frac{2}{6}\,(-18) = -6\,ms^{-1}$$

Collision 3 (B $\rightarrow$ C, where both bodies are moving!)



Relative velocities are reversed in an elastic collision

$$v_c - v_b = u_b - u_c = 12 - 9 = 3 \qquad (1)$$

And momentum is conserved

$$v_b + 5v_c = (1 \times 12) + (5 \times 9) = 57 \qquad (2)$$

From (1) & (2) $\quad v_b = 7 \quad v_c = 10$

Hence there are just 3 collisions and the final velocities are $v_a = -6\,ms^{-1} \quad v_b = 7\,ms^{-1} \quad v_c = 10\,ms^{-1}$

Initial kinetic energy

$$T_i = \tfrac{1}{2} m_b \, 27^2 = 364.5 \, J$$

Final kinetic energy

$$T_f = \tfrac{1}{2}\left(m_a \, 6^2 + m_b \, 7^2 + m_c \, 10^2 \right) = \tfrac{1}{2}(180 + 49 + 500) = 364.5 \, J$$

Hence kinetic energy is conserved in these elastic collisions as expected.
