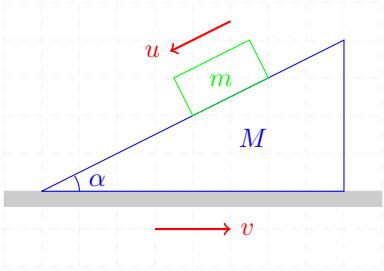
**Problem**

A wedge-shaped block of mass $M$ rests on a smooth horizontal table. A small block of mass $m$ is placed on its upper face, which is also smooth and inclined at an angle $\alpha$ to the horizontal. The system is released from rest. Write down the horizontal component of momentum, and the kinetic energy of the system, in terms of the velocity $v$ of the wedge and the velocity $u$ of the small block relative to it. Using conservation of momentum and the equation for the rate of change of kinetic energy, find the accelerations of the blocks. Given that $M = 1\,kg$ and $m = 250\,g$, find the angle $\alpha$ that will maximize the acceleration of the wedge.

**Solution**

Momentum and kinetic energy




$$\begin{aligned} Mv &= m(ucos\alpha - v) \\ v &= \frac{mucos\alpha}{M+m} \end{aligned}$$

$$\begin{aligned} T &= \tfrac{1}{2}Mv^2 + \tfrac{1}{2}m\big((v-ucos\alpha)^2 + (usin\alpha)^2\big) \\ &= \tfrac{1}{2}Mv^2 + \tfrac{1}{2}m(v^2 - 2uvcos\alpha + u^2) \\ &= \frac{M+m}{2}\left(\frac{mucos\alpha}{M+m}\right)^2 - mcos\alpha\left(\frac{mucos\alpha}{M+m}\right)u + \frac{m}{2}u^2 \\ &= \frac{m^2cos^2\alpha}{2(M+m)}u^2 - \frac{m^2cos^2\alpha}{M+m}u^2 + \frac{m}{2}u^2 \\ &= -\frac{m^2cos^2\alpha}{2(M+m)}u^2 + \frac{m}{2}u^2 \\ &= \frac{-m^2cos^2\alpha + m(M+m)}{2(M+m)}u^2 \\ &= \frac{m^2sin^2\alpha + mM}{2(M+m)}u^2 \end{aligned}$$

Rate of change of kinetic energy equation (rate at which gravity does work on mass $m$)

$$\begin{aligned} \dot{T} &= \dot{\mathbf{r}} \cdot \mathbf{F} \\ \frac{m^2sin^2\alpha + mM}{(M+m)}u\dot{u} &= u\,mgsin\alpha \\ \dot{u} &= \frac{(M+m)gsin\alpha}{m(msin^2\alpha + M)} \end{aligned}$$

$$\begin{aligned} \dot{v} &= \frac{mcos\alpha}{M+m}\dot{u} \\ &= \frac{mgsin\alpha cos\alpha}{msin^2\alpha + M} \\ &= \frac{mgsin2\alpha}{2(msin^2\alpha + M)} \end{aligned}$$

$M = 1\,kg$ and $m = 250\,g \quad \rightarrow \quad \dot{v} \propto sin2\alpha/(sin^2\alpha + 4)$

For maximum acceleration of the wedge $d\dot{v}/d\alpha = 0$, hence

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}\alpha}\left(\frac{sin2\alpha}{sin^2\alpha + 4}\right) &= 0 \\
(sin^2\alpha + 4)2cos2\alpha - sin2\alpha(2sin\alpha cos\alpha) &= 0 \\
(sin\alpha^2 + 4)(1 - 2sin^2\alpha) &= 2sin^2\alpha(1 - sin^2\alpha) \\
sin^2\alpha - 2sin^4\alpha + 4 - 8sin^2\alpha &= 2sin^2\alpha - 2sin^4\alpha \\
4 - 7sin^2\alpha &= 2sin^2\alpha \\
sin\alpha &= 2/3 \\
\alpha &= 41.8^\circ
\end{aligned}$$

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