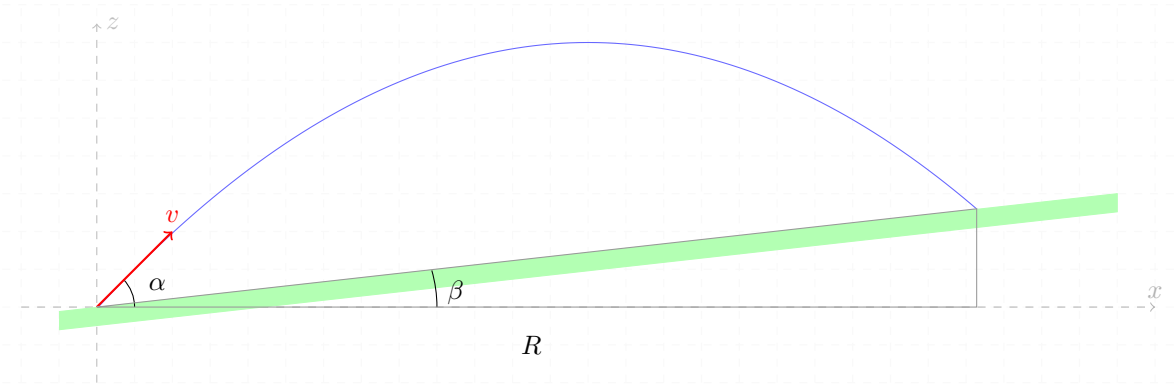
Problem

Find the equation for the trajectory of a projectile launched with velocity v at an angle α to the horizontal, assuming negligible atmospheric resistance. Given that the ground slopes at an angle β, show that the range of the projectile measured horizontally is

$$x = \frac{2v^2}{g}\frac{sin(\alpha-\beta)cos\alpha}{cos\beta}$$

At what angle should the the projectile be launched to achieve maximum range?

Solution



Equations of motion

$$\begin{aligned} x &= vcos\alpha t \qquad\qquad \rightarrow \quad t = x/vcos\alpha \\ z &= vsin\alpha t - \tfrac{1}{2}gt^2 \\ z &= xtan\alpha - \frac{gx^2}{2v^2cos^2\alpha} \end{aligned}$$

Projectile hits ground when $z = xtan\beta$

$$\begin{aligned} xtan\beta &= xtan\alpha - \frac{gx^2}{2v^2cos^2\alpha} \\ R &= \frac{2v^2}{g}\left(tan\alpha - tan\beta\right)cos^2\alpha \\ &= \frac{2v^2}{g}\left(\frac{sin(\alpha-\beta)}{cos\alpha cos\beta}\right)cos^2\alpha \qquad \text{(trig identity)} \\ R &= \frac{2v^2}{g}\frac{sin(\alpha-\beta)cos\alpha}{cos\beta} \end{aligned}$$

For maximum range $\partial R/\partial\alpha = 0$

$$\begin{aligned} \frac{\partial}{\partial\alpha}\left(sin(\alpha-\beta)cos\alpha\right) &= 0 \\ \frac{\partial}{\partial\alpha}\left(sin(2\alpha-\beta)+sin(-\beta)\right) &= 0 \\ cos(2\alpha-\beta) &= 0 \\ 2\alpha-\beta &= \pi/2 \\ \alpha &= \pi/4+\beta/2 \end{aligned}$$