Problem

Show that in the limit of strong damping (large γ) the time of flight of a projectile (on level ground) is approximately $t \approx (w/g + 1/\gamma)(1 - e^{-1-\gamma w/g})$. Show that to the same order of accuracy the range is $x \approx (u/\gamma)(1 - e^{-1-\gamma w/g})$. For a projectile launched at 800 ms^{-1} with $\gamma = 0.1\, s^{-1}$, estimate the range for launch angles of 30°, 20° and 10°.

Solution

Equations of motion for a projectile velocity $\mathbf{v}(u, 0, w)$ subject to a damping force, such as atmospheric drag (§3.2 in the text)

$$x = \frac{u}{\gamma}\left(1 - e^{-\gamma t}\right)$$
$$z = \left(\frac{w}{\gamma} + \frac{g}{\gamma^2}\right)\left(1 - e^{-\gamma t}\right) - \frac{gt}{\gamma}$$

At the end of the flight on level ground $z = 0$, hence from the second equation

$$t = \left(\frac{w}{g} + \frac{1}{\gamma}\right)\left(1 - e^{-\gamma t}\right)$$
$$\approx \left(\frac{w}{g} + \frac{1}{\gamma}\right) \qquad (\gamma t \text{ large})$$

Subs this value back into the original equation gives the approximate flight time

$$t' \approx \left(\frac{w}{g} + \frac{1}{\gamma}\right)\left(1 - e^{-1-\gamma w/g}\right)$$

And similarly the approximate range

$$x' \approx \frac{u}{\gamma}\left(1 - e^{-1-\gamma w/g}\right)$$

For $\gamma = 0.1$, $\quad v = 800\,(u = 800 cos\alpha \quad w = 800\, sin\alpha)$ estimated ranges

a) $\alpha = 30°$

$$x' \approx \frac{800 cos 30}{0.1}\left(1 - e^{-1-(0.1\times 800 sin 30)/9.81}\right) = 6928(.994) = 6.89\, km$$

b) $\alpha = 20°$

$$x' \approx \frac{800 cos 20}{0.1}\left(1 - e^{-1-(0.1\times 800 sin 20)/9.81}\right) = 7518(.977) = 7.35\, km$$

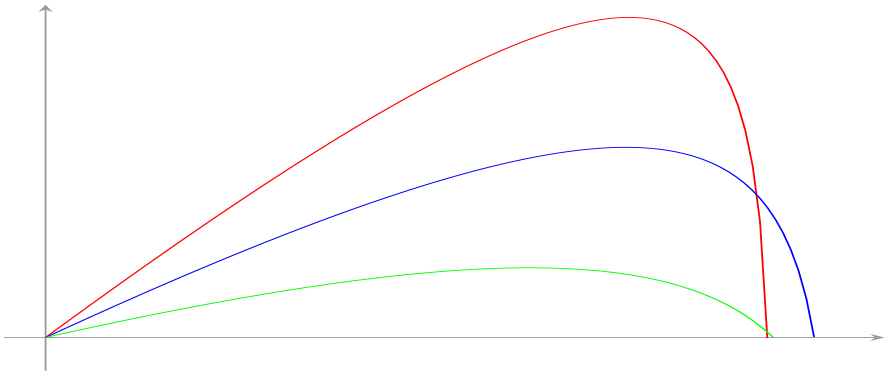
c) $\alpha = 10°$

$$x' \approx \frac{800 cos 10}{0.1}\left(1 - e^{-1-(0.1\times 800 sin 10)/9.81}\right) = 7878(.911) = 7.18\, km$$

Eliminating time from the original equations (see text)

$$z = \frac{\gamma w + g}{\gamma u} x + \frac{g}{\gamma^2} ln\left(1 - \frac{\gamma x}{u}\right)$$

A plot of z for launch angles $\alpha = 30°$ $\alpha = 20°$ $\alpha = 10°$



From the graph

a) $\alpha = 30°$ Range $= 6.88km$

b) $\alpha = 20°$ Range $= 7.33km$

c) $\alpha = 10°$ Range $= 6.94km$

The original estimates match these values closely except for the launch angle $\alpha = 10°$, which was somewhat over estimated.