Problem

An α-particle of energy $4\,keV$ ($1\,eV = 1.6 \times 10^{-19} J$) is scattered by an aluminium atom through an angle of 90°. Calculate the distance of closest approach to the nucleus. (Atomic number of α-particle =2, atomic number of Al=13, $e = 1.6 \times 10^{-19} C$.) A beam of such particles with a flux of $3 \times 10^8\, m^{-2} s^{-1}$ strikes a target containing $50\, mg$ of aluminium. A detector of cross-sectional area $400\, mm^2$ is placed $0.6\, m$ from the target in a direction at right angles to the beam direction. Find the rate of detection of α-particles. (Atomic mass of Al=27 u; $1u = 1.66 \times 10^{-27} kg$.)

Solution

Distance of closest approach r_1 (see § 4.3 in the text)

$$r_1 = a + \sqrt{a^2 + b^2}$$

Where

$$\begin{aligned} a &= \frac{qq'}{4\pi\epsilon_0 m v^2} \\ &= \frac{(2 \times 1.6 \times 10^{-19})(13 \times 1.6 \times 10^{-19})}{(4\pi \times 8.854 \times 10^{-12})(2 \times 4000 \times 1.6 \times 10^{-19})} \\ &= 4.67 \times 10^{-12}\, m \end{aligned}$$

And

$$b = a\, cot(\theta/2) = a\, cot(90/2) = a$$

Hence

$$\begin{aligned} r_1 &= a + \sqrt{a^2 + a^2} \\ &= a(1 + \sqrt{2}) \\ &= (4.67 \times 10^{-12})(2.414) \\ &= 1.13 \times 10^{-11}\, m \end{aligned}$$

The distance of closest approach to the nucleus is $1.13 \times 10^{-11}\, m$

Rate of detection (see §4.5 - §4.7 in the text)

$$dw = N\, f\, \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\, \frac{dA}{L^2} \quad s^{-1}$$

Where

N (no of atoms in target) $= (50 \times 10^{-6})/(27 \times 1.66 \times 10^{-27}) = 1.12 \times 10^{21}$
f (flux of α-particles) $= 3 \times 10^8\, m^{-2} s^{-1}$
$d\sigma/d\Omega$ (differential cross-section)$= a^2/4sin^4(\theta/2) = a^2/4sin^4(90/2) = a^2 = 2.18 \times 10^{-23}\, m^2 sr^{-1}$
dA (target area) $= 400 \times 10^{-12}\, m^2$
L (detector distance) $= 0.6\, m$

$$\begin{aligned} dw &= \frac{(1.12 \times 10^{21})(3 \times 10^8)(2.18 \times 10^{-23})(400 \times 10^{-12})}{(0.6)^2} \\ &= \frac{2.93 \times 10^{-3}}{0.36} = 8.14 \times 10^3\, s^{-1} \end{aligned}$$

The rate of detection of α-particles is $8.14 \times 10^3\, s^{-1}$.