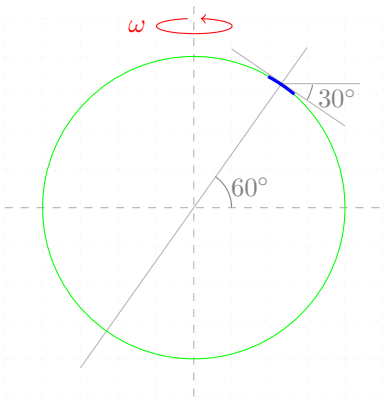
Problem

The water in a circular lake of radius $1\,km$ in latitude $60°$ is at rest relative to the Earth. Find the depth by which the centre is depressed relative to the shore by the centrifugal force. For comparison, find the height by which the centre is raised by the curvature of the Earth's surface.

Solution

The lake is subject to the horizontal component (i.e. tangential to the Earth's surface) of the pseudo centrifugal force. It has an effective centrifugal acceleration of $\rho(\omega cos30°)^2$ where ρ is the radius of the lake. The surface of the lake is an equipotential corresponding to the potential energy



$$V = -\tfrac{1}{2}m\rho^2(\omega cos30)^2$$

Hence, taking z as the amount by which the centre of the lake is depressed

$$\begin{aligned} gz &= \tfrac{1}{2}\rho^2\omega^2 cos^2 30° \\ z &= \frac{\rho^2\omega^2(.75)}{2g} \\ &= \frac{(10^3)^2(7.292\times10^{-5})^2(0.75)}{2(9.81)} \\ &= 2.032\times10^{-4}\,m \end{aligned}$$

The effect of the centrifugal force is to depress the centre of the lake by $0.20\,mm$.

Using the property of intersecting chords, the centre is raised by h, where

$$\begin{aligned} \rho\times\rho &= h(2R_E - h) \\ h &= \frac{\rho^2}{2R_E} \qquad\qquad h << R \\ &= \frac{(10^3)^2}{2(6.378\times10^6)} \\ &= 7.84\times10^{-2}\,m \end{aligned}$$

By way of comparison the height of the centre of the lake is raised $78\,mm$ by the Earth's curvature.
