Problem

A double star is formed of two components, each with mass equal to that of the Sun. The distance between them is 1 AU. What is the orbital period?

Solution

In CM frame with μ= reduced mass; r=distance between stars; m=mass of each star ($=M_s$)

$$\mu\ddot{\mathbf{r}} = \frac{Gm^2}{r^2}\,\hat{\mathbf{r}}$$

Since $\mu = \dfrac{m^2}{m+m} = \dfrac{m^2}{M}$

$$\mu\ddot{\mathbf{r}} = \frac{GM\mu}{r^2}\,\hat{\mathbf{r}}$$

Thus the stars orbit in ellipses around the centre of mass with period τ given by Kepler's Law.

$$\begin{aligned}
\left(\frac{\tau}{2\pi}\right)^2 &= \frac{r^3}{GM} \\
&= \frac{(1AU)^3}{G(2M_s)} \\
&= \frac{(1.496\times10^{11})^3}{(6.673\times10^{-11})(2\times1.989\times10^{30})} \\
\left(\frac{\tau}{2\pi}\right) &= 3.55\times10^6 \\
\tau &= 2.23\times10^7 s
\end{aligned}$$

The orbital period of the stars is 2.23×10^7 seconds or approximately 258 days.