Problem

Find the Lab differential cross-section for the scattering of identical particles of charge e and mass m, if the incident velocity is v.

Solution

The easiest way to find the Lab cross-section is to compute the CM cross-section and use the relation

$$\frac{\mathrm{d}\Omega^*}{\mathrm{d}\Omega} = \frac{sin\theta^* d\theta^*}{sin\theta d\theta} = \frac{\mathrm{d}z^*}{\mathrm{d}z} \quad \text{with} \quad z = cos\theta,\ z^* = cos\theta^*$$

Then for equal masses the relationship between the Lab and CM scattering angle

$$tan\theta = \frac{sin\theta^*}{(m_1/m_2) + cos\theta^*} \quad \rightarrow \quad z^* = 2z^2 - 1$$

Also for equal masses $\theta = \theta^*/2$ and $\alpha = 90 - \theta$

(See §7.3 & §7.4 in the text.)

In the CM frame each particle has an impact parameter of $b^*(b/2)$ and velocity $v^*(v/2)$, hence in this frame the impact parameter is related to the scattering angle (see §4.4 - §4.7 in the text)

$$\begin{aligned}
2b^* &= 2a^* cot(\theta^*/2) \quad \text{where} \quad a^* = \frac{e^2}{4\pi\epsilon_0 m(v^* + v^*)^2} \\
b &= acot(\theta^*/2) \qquad \text{where} \quad a = \frac{e^2}{2\pi\epsilon_0 m v^2} \\
|db| &= \frac{ad\theta^*}{2sin^2(\theta^*/2)^2} \\
d\sigma &= b|db|d\varphi \\
&= \frac{a^2 cos(\theta/2) d\theta^* d\varphi}{2sin^3(\theta^*/2)} \\
\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega^*} &= \frac{a^2}{4sin^4(\theta^*/2)} \quad \text{since} \quad d\Omega^* = sin\theta^* d\theta^* d\varphi \\
&= \frac{a^2}{4sin^4\theta}
\end{aligned}$$

The CM differential cross-section for incident particles is $a^2/4sin^4\theta$

In the Lab frame the differential cross-section for incident particles

$$\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} &= \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega^*}\frac{\mathrm{d}z^*}{\mathrm{d}z} \\
&= \frac{a^2}{4sin^4\theta} 4z \\
&= \frac{a^2 cos\theta}{sin^4\theta}
\end{aligned}$$

The cross-section for the recoiling particles is therefore $a^2 cos\theta/sin^4\alpha$ and the total cross-section

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{a^2 cos\theta}{sin^4\theta} + \frac{a^2 cos\theta}{sin^4\alpha} = a^2 cos\theta \left(\frac{1}{sin^4\theta} + \frac{1}{cos^4\theta}\right) \quad \text{where} \quad a = \frac{e^2}{2\pi\epsilon_0 m v^2}$$