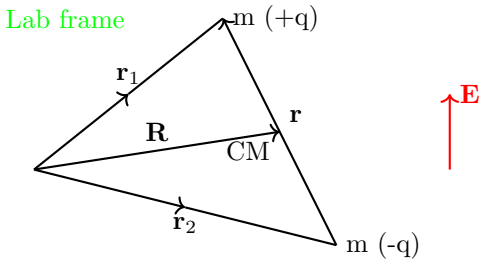
Problem

Write down the equations of motion for a pair of charged particles of equal masses m, and of charges q and -q, in a uniform electric field $\boldsymbol{E}$. Show that the field does not affect the motion of the centre of mass. Suppose that the particles are moving in circular orbits with angular velocity ω in planes parallel to the xy-plane, with $\boldsymbol{E}$ in the z-direction. Write the equations in a frame rotating with angular velocity ω, and hence find the separation of the planes.

Solution

Equations of motion



$$m\ddot{\mathbf{r}}_1 = q\mathbf{E} - \frac{q^2}{4\pi\epsilon_0 r^2}\,\hat{\mathbf{r}}$$

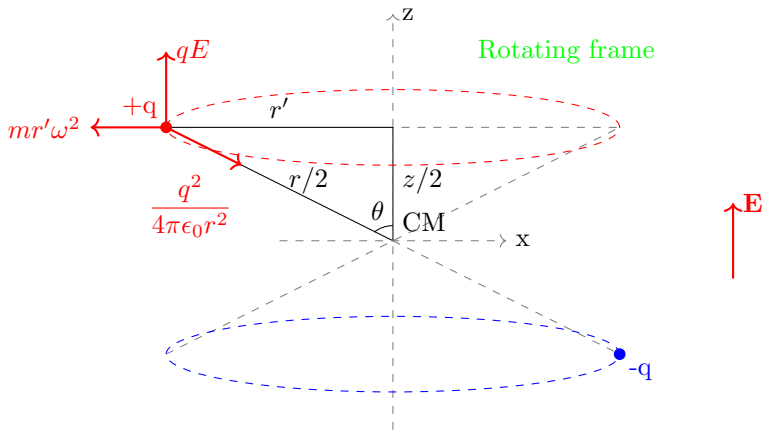
$$m\ddot{\mathbf{r}}_2 = -q\mathbf{E} + \frac{q^2}{4\pi\epsilon_0 r^2}\,\hat{\mathbf{r}}$$

Also

$$\mathbf{R} = \frac{m\mathbf{r}_1 + m\mathbf{r}_2}{m+m} = (\mathbf{r}_1 + \mathbf{r}_2)/2$$

Adding the equations of motion.

$m(\ddot{\mathbf{r}}_1 + \ddot{\mathbf{r}}_2) = 0$ or $2m\ddot{\mathbf{R}} = 0$ i.e. electric field does not affect motion of CM.



Resolving forces in the x and z directions respectively

$$mr'\omega^2 = \frac{q^2}{4\pi\epsilon_0 r^2}\,sin\theta \quad \rightarrow \quad \frac{m\omega^2}{2} = \frac{q^2}{4\pi\epsilon_0 r^3} \quad \text{since} \quad r' = \frac{r}{2}sin\theta$$

$$qE = \frac{q^2}{4\pi\epsilon_0 r^2}\,\cos\theta \quad \rightarrow \quad \frac{qE}{z} = \frac{q^2}{4\pi\epsilon_0 r^3} \quad \text{since} \quad cos\theta = \frac{z}{r}$$

Hence, equating these results $\dfrac{m\omega^2}{2} = \dfrac{qE}{z}$ and $z = \dfrac{2qE}{m\omega^2}$

The separation of the planes is $\dfrac{2qE}{m\omega^2}$.