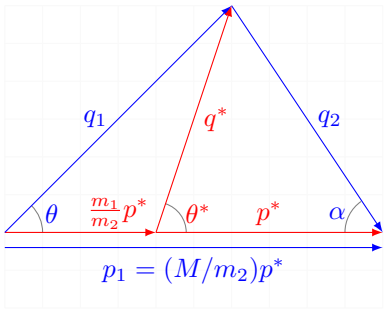
Problem

An experiment is to be designed to measure the differential cross-section for elastic pion-proton scattering at a CM scattering angle of 70° and a pion CM kinetic energy of 490 KeV. Find the angles in the Lab at which the scattered pions, and the recoiling protons, should be detected. and the required Lab kinetic energy of the pion beam. (The ratio of pion to proton mass is 1/7.)

Solution

For elastic collisions of this type the relationships between the momenta in the Lab frame and the CM frame(*) are conveniently summarized in this vector diagram. (See §7.3 in the text.)



Scattering angle α of pions (note: $p^* = q^*$)

$$\begin{aligned}\alpha &= \tfrac{1}{2}(180 - \theta^*) \\ &= \tfrac{1}{2}(180 - 70) \\ \alpha &= 55^\circ\end{aligned}$$

Recoiling protons (dropping perpendicular from vertex of triangle)

$$\begin{aligned}tan\theta &= \frac{q^* sin\theta^*}{\frac{m_1}{m_2}p^* + q^* cos\theta^*} \\ &= \frac{sin\theta^*}{m_1/m_2 + cos\theta^*} \\ &= \frac{sin70}{1/7 + cos70} = \frac{0.9397}{0.4849} = 1.938 \\ \theta &= 62.7^\circ\end{aligned}$$

Required Lab kinetic energy

$$\begin{aligned}T &= \frac{p_1^2}{2m_1} \\ &= \frac{M^2 p^{*2}}{2m_1 m_2^2} \\ &= \left(\frac{M}{m_2}\right)^2 \left(\frac{q^{*2}}{2m_1}\right) \\ &= \left(\frac{8}{7}\right)^2 T^* = \left(\frac{8}{7}\right)^2 490 \\ T &= 640\, KeV\end{aligned}$$