Problem

An unstable particle of mass $M = m_1 + m_2$ decays into two particles of masses m_1 and m_2, releasing an amount of energy Q. Determine the kinetic energies of the two particles in the CM frame. Given that $m_1/m_2 = 4$, $Q = 1\,MeV$, and that the unstable particle is moving in the Lab with kinetic energy $2.25\,MeV$, find the maximum and minimum Lab kinectic energies of the particle of mass m_1.

Solution

In the CM frame

$$T^* = \frac{q^{*\,2}}{2\mu} \quad \rightarrow \quad q^{*\,2} = 2\mu Q \qquad \text{where} \qquad \mu = \frac{m_1 m_2}{m_1 + m_2}$$

Hence

$$T_1^* = \frac{q^{*\,2}}{2m_1} = \frac{\mu Q}{m_1} = \frac{m_2}{M}\,Q \qquad \text{where} \qquad M = m_1 + m_2$$
$$T_2^* = \frac{q^{*\,2}}{2m_2} = \frac{\mu Q}{m_2} = \frac{m_1}{M}\,Q$$

$$\tfrac{1}{2} m_1 \dot{r}_1^2 = \frac{m_2}{M}\,Q \quad \rightarrow \quad \dot{r}_1 = \left(\frac{m_2}{m_1}\frac{2Q}{M}\right)^{1/2} = \tfrac{1}{2}\sqrt{\frac{2Q}{M}}$$
$$\tfrac{1}{2} m_2 \dot{r}_2^2 = \frac{m_1}{M}\,Q \quad \rightarrow \quad \dot{r}_2 = \left(\frac{m_1}{m_2}\frac{2Q}{M}\right)^{1/2} = 2\sqrt{\frac{2Q}{M}}$$

Conservation of momentum $\rightarrow$ relative velocity $\dot{r} = \dot{r_1} + \dot{r}_2 = \frac{5}{2}\sqrt{2/M}$

In the Lab frame

$$\tfrac{1}{2} M \dot{R}^2 = 2.25$$
$$\dot{R} = \tfrac{3}{2}\sqrt{2/M}$$

Max and Min KE are when $\dot{r}$ and $\dot{R}$ are in the same or opposite directions respectively.

Hence

$$\dot{r}_1 = \dot{R} \pm \frac{m_2}{M}\dot{r}$$
$$= \tfrac{3}{2}\sqrt{2/M} \pm \tfrac{1}{2}\sqrt{2/M}$$
$$\dot{r}_1 = 2\sqrt{2/M} \quad \text{or} \quad \sqrt{2/M}$$

$$KE_{Max} = \tfrac{1}{2} m_1 \dot{r}_1^2 = \frac{m_1}{2}\frac{8}{M} = \frac{32}{10} = 3.2$$
$$KE_{Min} = \frac{m_1}{2}\frac{2}{M} = \frac{8}{10} = 0.8$$

The maximum and minimum kinetic energies of m_1 in the Lab are $3.2\,MeV$ and $0.8\,MeV$ respectively.