Problem

Find the lengths of the 'day' and the 'month' (a) when the Moon was 10 Earth radii away, and (b) when the solar and lunar tides become equal in magnitude.

Solution

(a) The angular rotational velocity of the Earth ω is related to its current value ω_0 by equation 8.27

$$\begin{aligned}\frac{\omega}{\omega_0} &= 5.88 - 0.629\left(\frac{a_M}{R_E}\right)^{1/2} \\ &= 5.88 - 0.629(10)^{1/2} = 3.891 \\ \frac{\tau_0}{\tau} &= 3.891 \\ \tau &= 1/3.891 \quad \text{d} \\ &= 6.168 \quad \text{h}\end{aligned}$$

The length of the 'day' was 6.17 hours.

The Moon's current orbital period τ_M is $2.361 \times 10^6\, s$ and its semi-major axis a_M is $3.844 \times 10^5\, km$ or 60.34 Earth radii. Since $(\tau_M/2\pi)^2 = a^3/GM_E$ (Kepler's Law)

$$\begin{aligned}\left(\frac{\tau_M}{\tau}\right)^2 &= \left(\frac{60.34}{10}\right)^3 = 219.7 \\ \tau &= (2.361 \times 10^6)/14.82 \quad \text{s} \\ &= 1.593 \times 10^5 \quad \text{s} \\ &= 1.844 \quad \text{d}\end{aligned}$$

The length of the 'month' was 1.84 days.

(b) The solar and lunar tides become equal when the Moon is 78.2 Earth radii away. (This was calculated in a previous problem using the formula for tide height $h_0 = mr^4/Ma^4$. See Chapter 6 question 16.)

$$\begin{aligned}\frac{\omega}{\omega_0} &= 5.88 - 0.629\left(\frac{a_M}{R_E}\right)^{1/2} \\ &= 5.88 - 0.629(78.20)^{1/2} = 0.3177 \\ \frac{\tau_0}{\tau} &= 0.3177 \\ \tau &= 1/0.3177 \quad = 3.148 \quad \text{d}\end{aligned}$$

The length of the 'day' will be 3.15 days.

$$\begin{aligned}\left(\frac{\tau_M}{\tau}\right)^2 &= \left(\frac{60.34}{78.20}\right)^3 = 0.4594 \\ \tau &= (2.361 \times 10^6)/0.6778 \quad \text{s} \\ &= 3.483 \times 10^6 \quad \text{s} \\ &= 40.32 \quad \text{d}\end{aligned}$$

The length of the 'month' will be 40.3 days.