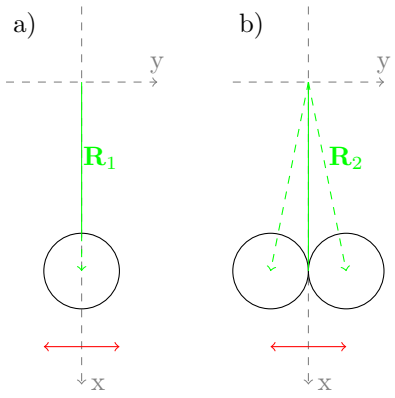
Problem

A pendulum consists of a light rigid rod of length $250\,mm$, with two identical uniform solid spheres of radius $50\,mm$ attached one on either side of its lower end. Find the period of small oscillations (a) perpendicular to the line of centres, and (b) along it.

Solution

a) The moment of inertia of a sphere mass M, radius a about a principal axis is $\frac{2}{5}Ma^2$. Hence the total moment of inertia of the system about the z-axis is given by



$$\begin{aligned} I &= 2(\tfrac{2}{5}Ma^2) + 2MR_1^2 \\ &= 2M\left(\tfrac{2}{5}(0.05)^2 + (0.25)^2\right) \\ &= 2M(0.0635) \end{aligned}$$

The period of small oscillations is given by

$$\begin{aligned} \tau &= 2\pi\sqrt{\frac{I}{(2M)gR_1}} \\ &= 2\pi\sqrt{\frac{0.0635}{9.81 \times 0.25}} \\ &= 1.011\,s \end{aligned}$$

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b) In this case the total moment of inertia is given by

$$\begin{aligned} I &= 2(\tfrac{2}{5}Ma^2) + 2MR_2^2 \\ &= 2M\left(\tfrac{2}{5}(0.05)^2 + (0.25^2 + 0.05^2)\right) \\ &= 2M(0.066) \end{aligned}$$

And the period of small oscillations is given by

$$\begin{aligned} \tau &= 2\pi\sqrt{\frac{I}{(2M)gR_1}} \\ &= 2\pi\sqrt{\frac{0.066}{9.81 \times 0.25}} \\ &= 1.031\,s \end{aligned}$$
