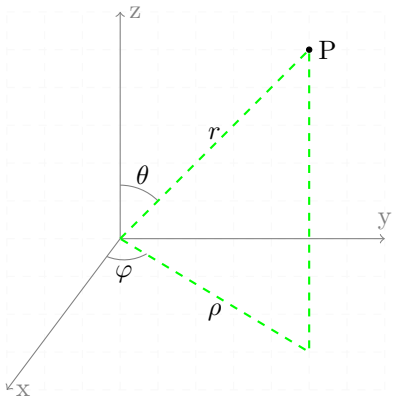
In common physics notation, θ is the polar angle from the z-axis to the radial vector $\mathbf{r}$, φ the azimuthal angle from the x-axis to the projection(ρ) of $\mathbf{r}$ on the xy-plane.



Cartesian (x, y, z)

Vector notation

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \qquad \dot{\mathbf{r}} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j} + \dot{z}\mathbf{k}$$

Volume element

$$dV = dx\,dy\,dz$$

Langrangian (Central Conservative Force)

$$L = \tfrac{1}{2}m\dot{\mathbf{r}}^2 - V = \tfrac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - V(x, y, z)$$

Cylindrical Polars (ρ, φ, z)

$$\begin{pmatrix}\rho\\ \varphi\\ z\end{pmatrix} = \begin{pmatrix}\sqrt{x^2+y^2}\\ arctan(y/x)\\ z\end{pmatrix} \qquad \begin{pmatrix}x\\ y\\ z\end{pmatrix} = \begin{pmatrix}\rho cos\\ \rho sin\\ z\end{pmatrix}$$

Vector notation

$$\begin{pmatrix}\hat{\boldsymbol{\rho}}\\ \hat{\boldsymbol{\varphi}}\\ \mathbf{k}\end{pmatrix} = \begin{pmatrix}cos\varphi & sin\varphi & 0\\ -sin\varphi & cos\varphi & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\mathbf{i}\\ \mathbf{j}\\ \mathbf{k}\end{pmatrix} \qquad \begin{pmatrix}\mathbf{i}\\ \mathbf{j}\\ \mathbf{k}\end{pmatrix} = \begin{pmatrix}cos\varphi & -sin\varphi & 0\\ sin\varphi & cos\varphi & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}\hat{\boldsymbol{\rho}}\\ \hat{\boldsymbol{\varphi}}\\ \mathbf{k}\end{pmatrix}$$

$$\mathbf{r} = \rho\,\hat{\boldsymbol{\rho}} + z\mathbf{k} \qquad \dot{\mathbf{r}} = \dot{\rho}\,\hat{\boldsymbol{\rho}} + \rho\dot{\varphi}\,\hat{\boldsymbol{\varphi}} + \dot{z}\mathbf{k}$$

Volume element

$$dV = \rho\,d\rho\;d\varphi\,dz$$

Langrangian

$$L = \tfrac{1}{2}m\dot{\mathbf{r}}^2 - V = \tfrac{1}{2}m(\dot{\rho}^2 + \rho^2\dot{\varphi}^2 + \dot{z}^2) - V(\rho, \varphi, z)$$

Spherical Polars (r, θ, φ)

$$\begin{pmatrix}r\\ \theta\\ \varphi\end{pmatrix} = \begin{pmatrix}\sqrt{x^2+y^2+z^2}\\ arccos(z/r)\\ arctan(y/x)\end{pmatrix} \qquad \begin{pmatrix}x\\ y\\ z\end{pmatrix} = \begin{pmatrix}rsin\theta cos\varphi\\ rsin\theta sin\varphi\\ rcos\theta\end{pmatrix}$$

Vector notation

$$\begin{pmatrix}\hat{\boldsymbol{r}}\\ \hat{\boldsymbol{\theta}}\\ \hat{\boldsymbol{\varphi}}\end{pmatrix} = \begin{pmatrix}sin\theta cos\varphi & sin\theta sin\varphi & cos\theta\\ cos\theta cos\varphi & cos\theta sin\varphi & -sin\theta\\ -sin\varphi & cos\varphi & 0\end{pmatrix}\begin{pmatrix}\mathbf{i}\\ \mathbf{j}\\ \mathbf{k}\end{pmatrix} \qquad \begin{pmatrix}\mathbf{i}\\ \mathbf{j}\\ \mathbf{k}\end{pmatrix} = \begin{pmatrix}sin\theta cos\varphi & cos\theta cos\varphi & -sin\theta\\ sin\theta sin\varphi & cos\theta sin\varphi & cos\varphi\\ cos\theta & -sin\theta & 0\end{pmatrix}\begin{pmatrix}\hat{\boldsymbol{r}}\\ \hat{\boldsymbol{\theta}}\\ \hat{\boldsymbol{\varphi}}\end{pmatrix}$$

$$\mathbf{r} = r\,\hat{\boldsymbol{r}} \qquad \dot{\mathbf{r}} = \dot{r}\,\hat{\boldsymbol{r}} + r\dot{\theta}\,\hat{\boldsymbol{\theta}} + rsin\theta\dot{\varphi}\,\hat{\boldsymbol{\varphi}}$$

Volume element

$$dV = r^2 sin\theta\,d\theta\;d\varphi\,dr$$

Langrangian

$$L = \tfrac{1}{2}m\dot{\mathbf{r}}^2 - V = \tfrac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + r^2 sin^2\theta\dot{\varphi}^2) - V(r, \theta, \varphi)$$