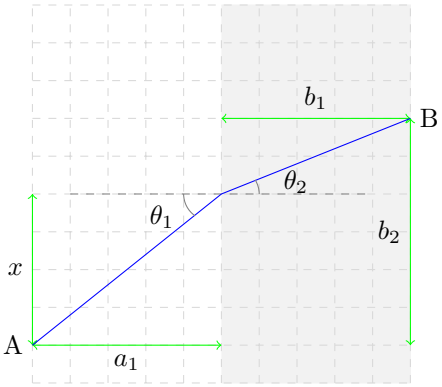
Exercise 1

Use Fermat's principle of least time to derive Snell's Law ($n_1 \sin\theta_1 = n_2 \sin\theta_2$).

Solution

Time taken to travel from A in medium 1 (index n_1, speed c/n_1)
to B in medium 2 (index n_2, speed c/n_2)



$$t = \frac{n_1\sqrt{a_1^2 + x^2}}{c} + \frac{n_2\sqrt{b_1^2 + (b_2 - x)^2}}{c}$$

Fermat's principle of least time $\dfrac{\mathrm{d}t}{\mathrm{d}x} = 0$

$$\frac{n_1 x}{\sqrt{a_1^2 + x^2}} - \frac{n_2(b_2 - x)}{\sqrt{b_1^2 + (b_2 - x)^2}} = 0$$

Hence

$$n_1 \frac{x}{\sqrt{a_1^2 + x^2}} = n_2 \frac{(b_2 - x)}{\sqrt{b_1^2 + (b_2 - x)^2}}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$
