Exercise 4

The Lagrangian for a free particle is $L = -mc^2/\gamma$. Find an expression for L, p and H when $v << c$.

Solution

$$\begin{aligned} L &= -\frac{mc^2}{\gamma} \\ &= -mc^2\left(1-\frac{v^2}{c^2}\right)^{1/2} \\ &\approx -mc^2\left(1-\frac{v^2}{2c^2}\right) \qquad\qquad (v << c) \\ &= -mc^2 + \frac{mv^2}{2} \end{aligned}$$

$$\begin{aligned} p &= \frac{\partial L}{\partial v} \\ &= mv \end{aligned}$$

$$\begin{aligned} H &= 2T - L \\ &= mv^2 - \left(-mc^2 + \frac{mv^2}{2}\right) \\ &= mc^2 + \frac{mv^2}{2} \end{aligned}$$
