Exercise 3

Show that the equations of motion following from the Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_1)^2 - \frac{1}{2}m^2\phi_1^2 + \frac{1}{2}(\partial_\mu \phi_2)^2 - \frac{1}{2}m^2\phi_2^2 - g(\phi_1^2 + \phi_2^2)^2$$

are the coupled equations

$$\partial_\mu \partial^\mu \phi_1 + m^2\phi_1 + 4g\phi_1(\phi_1^2 + \phi_2^2) = 0$$
$$\partial_\mu \partial^\mu \phi_2 + m^2\phi_2 + 4g\phi_2(\phi_1^2 + \phi_2^2) = 0$$

Solution

For ϕ_1

$$\partial\mathcal{L}/\partial\phi_1 = -m^2\phi - 4g\phi_1(\phi_1^2 + \phi_2^2) \qquad \partial\mathcal{L}/\partial(\partial_\mu\phi_1) = \partial^\mu\phi_1$$

Euler Lagrange

$$\frac{\partial\mathcal{L}}{\partial\phi_1} - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi_1)}\right) = 0$$
$$-m^2\phi_1 - 4g\phi_1(\phi_1^2 + \phi_2^2) - \partial_\mu\partial^\mu\phi_1 = 0$$
$$\partial_\mu\partial^\mu\phi_1 + m^2\phi_1 + 4g\phi_1(\phi_1^2 + \phi_2^2) = 0$$

And similarly for ϕ_2

Euler Lagrange

$$\partial_\mu\partial^\mu\phi_2 + m^2\phi_2 + 4g\phi_2(\phi_1^2 + \phi_2^2) = 0$$